# I B.Tech I Semester Supplementary Examinations, Feb/Mar 2014 MATHEMATICS-I <br> ( Common to Civil Engineering, Electrical \& Electronics Engineering, Mechanical Engineering, Electronics \& Communication Engineering, Computer Science \& Engineering, Chemical Engineering, Electronics \& Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics \& Computer Engineering, Aeronautical Engineering, Bio-Technology, Automobile Engineering, Mining and Petroliem Technology) 

Time: 3 hours
Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks

1. (a) Solve $\left(x^{2}+y^{2}-a^{2}\right) x d x+\left(x^{2}-y^{2}-b^{2}\right) y d y=0$.
(b) If air is maintained at $20^{\circ} \mathrm{C}$ and the temperature of the body cools from $80^{\circ} \mathrm{C}$ to $60^{\circ} \mathrm{C}$ in 10 minutes, find the temperature of the body after 30 minutes.
2. (a) Solve $\left(D^{2}+a^{2}\right) y=$ Sec ax
(b) Solve $\left(D^{2}+4\right) y=e^{x}+\operatorname{Sin} 2 x$
3. (a) If $\mathrm{V}=\log \left(x^{2}+y^{2}\right)+x-2 y$ find $\frac{\partial V}{\partial x} \cdot, \frac{\partial V}{\partial y}, \frac{\partial^{2} V}{\partial x^{2}} \cdot \frac{\partial^{2} V}{\partial y^{2}}$.
(b) If $\mathrm{U}=x e^{x y}$ where $x^{2}+y^{2}+2 x y=1, \quad$ find $\frac{\partial^{2} U}{\partial x^{2}}$.
4. (a) Trace the curve $\mathrm{r}=2+3 \sin \theta$.
(b) Trace the curve $y^{2}(2 a-x)=x^{3}$.
5. (a) Find the surface of the solid generated by revolution of the lemniscate $r^{2}=$ $a^{2} \cos ^{2} \theta$ about the initial line.
(b) Show that the whole length of the curve $x^{2}\left(a^{2}-x^{2}\right)=8 a^{2} y^{2}$ is $\pi a \sqrt{2}$. $\quad[8+7]$
6. (a) Show that $\int_{0}^{4 a} \int_{\frac{y^{2}}{4 a}}^{y} \frac{x^{2}-y^{2}}{x^{2}+y^{2}} \mathrm{dx} \mathrm{dy}=8 a^{2}\left(\frac{\pi}{2}-\frac{5}{3}\right)$.
(b) Evaluate $\iint_{R} y d x d y$ where R is the domain bounded by y -axis, the curve $\mathrm{y}=x^{2}$ and the line $x+y=2$ in the first quadrants.
7. (a) If $\mathrm{V}=\mathrm{e}^{x y z}(\mathrm{i}+\mathrm{j}+\mathrm{k})$, find curl V .
(b) Find the constants a and b so that the surface $\mathrm{ax}^{2}$-byz $=(\mathrm{a}+2) \mathrm{x}$ will be orthogonal to the surface $4 x^{2} y+z^{3}=4$ at the point $(1,-1,2)$
8. (a) Show that the area of the ellipse $x^{2} / a^{2}+y^{2} / b^{2}=1$ is $\pi a b$
(b) If $\mathrm{f}=\left(2 \mathrm{x}^{2}-3 \mathrm{z}\right) \mathrm{i}-2 \mathrm{xyj}-4 \mathrm{xzk}$, evaluate
(i) $\int_{v} \nabla \cdot f d V$ and
(ii) $\int_{v} \nabla \times f d V$ where V is the closed region bounded by $\mathrm{x}=0, \mathrm{y}=0, \mathrm{z}=0$, $2 \mathrm{x}+2 \mathrm{y}+\mathrm{z}=4$.

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1. (a) Solve $e^{y}\left(1+\frac{d y}{d x}\right)=e^{x}$
(b) Show that the family of curves $\frac{x^{2}}{a^{2}+\lambda}+\frac{y^{2}}{a^{2}+\lambda}=1$, where ' $\lambda$ ' is a parameter is self orthogonal.
$[8+7]$
2. (a) Solve $\left(D^{2}+9\right) y=2 \operatorname{Cos}^{2} x$.
(b) Solve $\frac{d^{2} y}{d x^{2}}+4 y=2 e^{x} \operatorname{Sin}^{2} x$.
$[8+7]$
3. (a) Calculate the approximate value of $\sqrt{10}$ to four decimal places using Taylor's theorem.
(b) Find 3 positive numbers whose sum is 600 and whose product is maximum.
4. (a) Trace the curve $y=x^{2}\left(x^{2}-4\right)$. (b)Trace the curve $\mathrm{r}=\cos \theta$. [8+7]
5. (a) The figure bounded by a parabola and the tangents at the extremities of its latusrectum revolves about the axis of the parabola, Find the volume of the solid thus generated.
$[8+7]$
(b) The segment of the parabola $\mathrm{y}^{2}=4 \mathrm{ax}$ which is cutoff by the latus rectum revolves about the directrix. Find the volume of rotation of the annular region.
6. (a) Evaluate $\iint(x+y)^{2} d x$ dy. over the area bounded by the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$.
(b) Transform the following to Cartesian form and hence evaluate $\int_{0}^{\pi} \int_{0}^{a} r^{3} \sin \theta d r d \theta$. $[8+7]$
7. (a) Prove that $\nabla \mathrm{r}=\bar{r} / \mathrm{r}$
(b) Find the angle between the surfaces $x^{2}+y^{2}+z^{2}=9$ and $z=x^{2}+y^{2}-3$ at the point ( $2,-1,2$ ).
8. (a) Evaluate $\iint_{S}(y z i+z x j+x y k) \cdot d S$ where S is the surface of the sphere $\mathrm{x}^{2}+\mathrm{y}^{2}+\mathrm{z}^{2}=\mathrm{a}^{2}$ in the first octant.
(b) Evaluate $\oint_{c}\left(x^{2}-2 x y\right) d x+\left(x^{2} y+3\right) d y$ around the boundary of the region defined by $\mathrm{y}^{2}=8 \mathrm{x}$ and $\mathrm{x}=2$.

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1. (a) Solve $y(\operatorname{Sin} x-y) d x=\operatorname{Cos} x d y$
(b) If the temperature of air is maintained at $20^{\circ} \mathrm{C}$ and the temperature of the body cools from $100^{\circ} \mathrm{C}$ to $80^{\circ} \mathrm{C}$ in 10 minutes, find the temperature of the body after 20 minutes.

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[8+7]
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2. (a) Solve $\left(D^{2}-4 D+13\right) y=e^{2 x}$
(b) Solve $\left(D^{2}-3 D+2\right) y=\operatorname{Cosh} x$
3. (a) If $\mathrm{r}+\mathrm{s}+\mathrm{t}=\mathrm{x}, \mathrm{s}+\mathrm{t}=\mathrm{xy}, \mathrm{t}=\mathrm{xyz}$, find $\frac{\partial(r, s, t)}{\partial(x, y, z)}$.
(b) Find the extreme points of $f(x, y)=x y+\frac{8}{x}+\frac{8}{y}$.
4. (a) Trace the curve $y=5 \cosh \left(\frac{x}{5}\right)$.
(b) Trace the curve $y^{2}=(4-x)\left(3-x^{2}\right) .$.
5. (a) Find the length of the arc of the curve $\mathrm{y}=\log (\sec x)$ from $x=o$ to $\frac{\pi}{3}$.
(b) Find the perimeter of the loop of the curve $3 \mathrm{ay}^{2}=\mathrm{x}(\mathrm{x}-\mathrm{a})^{2}$.
6. (a) Evaluate $\iint r d r d \theta$ over the region bounded by the cardioid $\mathrm{r}=\mathrm{a}(1+\cos \theta)$ and out side the circle $\mathrm{r}=\mathrm{a}$.
(b) Change the order of Integration \& evaluate $\int_{0}^{4 a} \int_{\frac{x^{2}}{4 a}}^{2 \sqrt{a x}} d y d x \quad[8+7]$
7. (a) Prove that $(\mathrm{F} \times \nabla) \times \bar{r}=-2 \mathrm{~F}$
(b) Determine the constant a so that the vector $\mathrm{V}=(\mathrm{x}+3 \mathrm{y}) \mathrm{i}+(\mathrm{y}-\mathrm{z}) \mathrm{j}+(\mathrm{x}+\mathrm{az}) \mathrm{k}$ is solenoidal.
8. Apply Stokes theorem, to evaluate $\oint_{c} y d x+z d y+x d z$ where C is the curve of intersection of the sphere $x^{2}+y^{2}+z^{2}=a^{2}$ and $x+z=a$.

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## Answer any FIVE Questions

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1. (a) Solve $(x+1) \frac{d y}{d x}-y=e^{3 x}(x+1)^{2}$
(b) Find the orthogonal trajectory of the family of curves $x^{2 / 3}+y^{2 / 3}=a^{2 / 3}$, where ' $a$ ' is a parameter
2. (a) Solve $\left(D^{3}-6 D^{2}+11 D-6\right) y=e^{-2 x}+e^{-3 x}$
(b) Solve $\frac{d^{2} y}{d x^{2}}-8 \frac{d y}{d x}+15 y=0$
3. (a) If $\mathrm{a}=\frac{y z}{x}, \mathrm{~b}=\frac{x z}{y}, \mathrm{c}=\frac{x y}{z}$, find $\frac{\partial(x, y, z)}{\partial(a, b, c)}$.
(b) Find the minimum value of $x^{2}+y^{2}+z^{2}$, give that $x y z=a^{3}$
4. (a) Trace the curve $\mathrm{r}=\cos 4 \theta$.
(b) Trace the curve $y^{2}(1-x)=x^{2}(1+x)$..
5. Prove that the volume of the solid generated by the revolution about the $x$-axis of the loop of the curve $x=t^{2}, y=t-\frac{1}{3} t^{3}$ is $\frac{3 \pi}{4}$.
$[8+7]$
6. (a) By changing the order of integration evaluate $\int_{0}^{1} \int_{0}^{\sqrt{2-x^{2}}} \frac{x}{\overline{x^{2}+y^{2}}} d y d x$.
(b) Evaluate $\int_{0}^{a} \int_{a-x}^{\sqrt[a^{2}-x^{2}]{ }} y d x$ dy by using change of order of integration. [8+7]
7. (a) If $\mathrm{V}=\mathrm{e}^{x y z}(\mathrm{i}+\mathrm{j}+\mathrm{k})$, find curl V .
(b) Find the constants $a$ and $b$ so that the surface $\mathrm{ax}^{2}$-byz $=(\mathrm{a}+2) \mathrm{x}$ will be orthogonal to the surface $4 x^{2} y+z^{3}=4$ at the point $(1,-1,2)$
$[8+7]$
8. (a) Use divergence theorem to evaluate $\iint_{S}\left(x^{3} i+y^{3} j+z^{3} k\right) . N d s$, and S is the surface of the sphere $x^{2}+y^{2}+z^{2}=r^{2}$.
(b) Using Green's theorem, Find the area bounded by the hypocycloid $x^{2 / 3}+y^{2 / 3}=$ $\mathrm{a}^{2 / 3}, \mathrm{a}>0$. Given that the parametric equations are $\mathrm{x}=\mathrm{a} \cos ^{3} \theta, \mathrm{y}=\mathrm{a} \sin ^{3} \theta$.
